DYNAMICS OF THE CEREBRAL CORTEX AUTOMATIC
DEVELOPMENT OF EQUILIBRIUM IN
SELF-ORGANIZING SYSTEMS

W. R. ASHBY
DEPARTMENT OF RESEARCH, BARNWOOD HOUSE
GLoucester, ENGLAND

The nervous system, and particularly the cerebral cortex, is examined in certain of its activities, it being treated as a purely physical dynamic system. It is shown mathematically that if, as seems likely, the cerebral cortex can undergo manifold changes of neurotic organization, then it follows that these changes must automatically lead to the development of more and more states of equilibrium, the process being unavoidable and largely irreversible. The theorem may be established on basic physical principles without appeal to special physiological details. The possible significance of this observation is indicated.

1.

It is generally accepted that the nervous system may be regarded as a purely physico-chemical or mechanistic system. This involves two assumptions:

(a) That its state may be specified by the numerical values at any moment of suitably selected variables. (The technical difficulty of actual measurement and the great number of them required do not affect the principle);

(b) That if we knew the state of every reacting part we could predict with certainty what it would do next; or in other words, its change at any moment depends only on its present state. (It will commonly be necessary to include environmental actions and reactions to make the system complete.)

This means (Note 1)* that the behavior of the system can be described by equations of the type

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n) \quad (i = 1, 2, \ldots, n).$$

Such a system of variables, i.e., whose fluxions are functions only of the variables of that system, t in particular being absent from the

* To avoid interrupting the line of argument with minor, but necessary, matters, these have been collected into "Notes" at the end of the paper.
right-hand side, is an "absolute" system. It is a well-known theorem that such a system of equations defines a congruence of curves in the corresponding \( n \)-dimensional space having co-ordinates \( x_1, x_2, \ldots, x_n \). This congruence, the "\( x \)-field," forms a representation of the behavior of the system which is very convenient for the later proofs (Note 2).

The "organization" of the system must now be identified with the set of \( f \)'s of equations (1). No other identification is possible.

It appears probable that some parts of the nervous system, particularly the cerebral cortex, can undergo internal reorganizations which result in a change of behavior. The many facts of learning and training indicate this. In view of the known variability of behavior in the higher animals, it seems that this postulate must be allowed; for if we hold to the "mechanistic" hypothesis we must allow that changed behavior must be the result of some inner, physical change (Note 3).

These "reorganizations" are the subject of this paper. It will be shown that they must always tend in the direction of increasing equilibrium.

2.

It will now be shown that a spontaneous change of organization implies the presence of a step-function of the time. A change of organization, by definition (§ 1), means that the functional forms \( f_i \) must change to \( \phi_i \), say. This change may be represented with equal generality as being due to a parameter \( h \), in the \( f \)'s, the \( h \) changing value from, say, \( h' \) to \( h'' \). This gives the two required organizations, i.e.,

\[
\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n; h')
\]

and

\[
\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n; h'').
\]

This change (from \( h' \) to \( h'' \)), however, by the "absoluteness" hypothesis (equation 1), is not to be arbitrary but is to depend on the internal events of the system. This means that \( h \) is to be treated as an extra variable in the system and that its change must be a function of given \( x, h \) combinations. \( h \), therefore, regarded as an ordinary variable of the system, will be observed in its time-development to be constant at \( h' \) for a time, and then to change to \( h'' \), where it continues. \( h \) must therefore be a step-function of the time (Note 4).

At this point it must be noted that the system composed of both
the $x$'s and $h$ is absolute and has one field in $n+1$ dimensions; and it has one and only one organization. This system must be clearly distinguished from the system composed of the $x$'s only, which is also absolute in the intervals while $h$ is constant, and in this interval it has one field in $n$ dimensions. But owing to the possibility of $h$-change, the $x$-system has two possible organizations. The effect of the change of $h$ is to change one $x$-field to another (Equations 2) (Note 5). This sudden change of a field from one form to another due to movement of the $x$-point in the field is fundamental here, and the concept will be used repeatedly in the paper. It is useless to proceed unless the reader is satisfied with the validity of this concept.

The presence of more than one step-function in the system adds further possible spontaneous changes of organization in the $x$-system. If there are $s$ step-functions $h$, each of which may take $r$ values, then the $x$'s will have $r^s$ organizations. (Each step-function must have defined the $x$ and $h$ points at which it changes value. These points will be referred to as $\theta$-points, from the equation in Note 4).

We now define a commutive system as one which (a) is absolute, (b) contains step-functions of the time $h_i$, (c) contains an infinite number of organizations of the variables $x$, provided by the step-functions $h$.

It is now easily shown that the initial hypotheses of § 1 are equivalent to: The nervous system and its environment form a commutive system (Note 6).

3.

We now proceed to study a peculiar property of commutive systems. A commutive system being absolute, its variables $x$ and $h$ will change with the time, the $h$'s change being discontinuous. (Some further properties are mentioned in Note 7). We now ask: Under what conditions can the $h$'s become permanently constant, in spite of repeated disturbances affecting the $x$'s? (Note 8) Constancy of the $h$'s is important since it corresponds to constancy of the $x$-field; and it is therefore the necessary and sufficient condition that the $x$'s should demonstrate a constant pattern of behavior after disturbance. An important sufficient condition is given by the following theorem: In any commutive system, if some external disturbance brings the $x$-point repeatedly to points $A, B, \ldots$ (at random times and in random order) a sufficient condition (Note 9) for subsequent permanent constancy of the $h$'s is that an $x$-field should occur with both the $x$-point at that moment and the points $A, B, \ldots$ all on paths which (a) after some finite time meet no new $x$-points and (b) meet no
\( \theta \)-points. \textbf{Proof}: On the occurrence of such a field, the \( x \)-point may continue to move as long as we please (Note 10) but no \( \theta \)-points can be met by the \( x \)-point, since it can only follow the paths from \( A, B, \ldots \) and these contain no \( \theta \)-points. And as change of field occurs only on the \( x \)-point meeting a \( \theta \)-point, change of field and of path cannot occur; so the \( x \)-point is permanently kept to these paths and is therefore permanently barred from meeting a \( \theta \)-point. So the \( h \)'s and the field are permanently constant.

A definition of "equilibrium" is now required. A path is defined here as "equilibrial" if, from some point onwards, it always remains within a fixed region of the \( x \)-space (Note 11). It is clear that the paths mentioned above (which meet no new \( x \)-points after a finite time) are equilibrial in the sense given. For after the finite time, the path must either come to an end, or join back on to itself forming a cycle; but in either case it remains within a finite region.

As there are an infinite number of fields we now proceed by using probability methods (Note 7). It will now be shown that: if there is a constant probability \( p \) that a field provided by a random combination of \( h \)-values in a commutive system will be of the type proved sufficient for \( h \)-constancy above (Note 12), then the probability \( P \) that the system after \( m \) field-changes will be \( h \)-constant is at least (Note 13)

\[
1 - (1 - p)^m.
\]

\textbf{Proof}: The occurrence of such a field being sufficient to cause \( h \)-constancy (here called "success"), and since a success stays so, while the failures at each field-change are given a fresh chance \( p \) of succeeding, we have (if \( 1 - p = q \)) that at the first trial \( p \) succeed and \( q \) fail; in the second trial the \( q \) will be divided so that \( q \) succeed and \( q \) fail; and so on. The proportion succeeding at the \( m \)-th trial will be \( pq^{m-1} \), and \( P \) will be at least

\[
p + pq + pq^2 + \cdots + pq^{m-1} = 1 - (1 - p)^m. \tag{3}
\]

In other words, \( m \) field-changes in a commutive system increase the probability of equilibrium from \( p \) to \( 1 - (1 - p)^m \), the latter always being greater. If we now add the trifling postulate that so long as the \( h \)'s are not constant \( m \to \infty \) as \( t \to \infty \), then clearly

\[
\lim_{t \to \infty} P = 1. \tag{4}
\]

\textbf{Equation (4) is the point of the whole paper}. It means that, in a commutive system, however small \( p \) may be, making \( m \) (or \( t \)) large
enough will ensure that $P$ approaches 1. In other words, no matter how rare equilibrial paths are in the individual $x$-fields, a commutive system will inevitably keep changing them until it does develop an equilibrium (Note 14). A commutive system is selective for organizations having equilibrium. And by our previous conclusion (that the nervous system and its environment form a commutive system) this must apply equally to the nervous system.

We see therefore that the assumption that the nervous system is a physical system and that it can undergo changes of organization leads without any further hypothesis (Note 15) to the deduction that it must be a commutive system ($\S$ 2), and this leads to the conclusion ($\S$ 3) that it must tend to develop an organization giving equilibrium, no other organization being capable of persisting.

5.

This paper is hardly concerned with applications, but two may be mentioned.

After the establishment of equilibrium has occurred, all disturbances (i.e., displacements towards $A, B, \ldots$) will be followed by activities of the system which will always oppose the disturbance (this being a characteristic of all systems in equilibrium).

This opposition to disturbance may, if the activity of the reaction is small, explain the phenomenon of “habituation.”

If the reaction to the disturbance is more active and complex, it may explain the development of “adaptive” behavior, since adaptation seems to be essentially the development of that organization which will preserve equilibrium (Notes 16, 17, 18, 19).

NOTES

(1) This can be proved in much finer detail, but the details are hardly required here. Its derivation from (b) is almost direct.

(2) A configuration of the system corresponds to a point in the space. The behavior of the system in time corresponds to the point moving along a path. All the paths make the field and this defines the organization of the system. “Experimental control” is equivalent to the ability to start the point where we like in the field (by using the $n$ arbitrary constants in the solution of equation 1).

(3) We do not need here to know anything of the physiological mechanism underlyng such change.

(4) To preserve the form of equations (1) we may use any continuous approximation to step-function form such as

$$\frac{dh}{dt} = q \left[ -\frac{h' + h''}{2} + \frac{h' - h''}{2} \tanh(\theta(x_1, \ldots, x_n; h)) - h \right];$$

where $q$ is positive and large. The function $\theta$ determines those $x$ and $h$ combinations where $h'$ changes to $h''$; for where $\theta(x_1, \ldots, x_n; h)$ is positive, $h$ becomes $h'$, while where it is negative $h$ changes to $h''$. 

NOTES
The whole \( z \)-field remains constant until \( h \) changes; then the \( z \)-field changes, in general to something quite different. Consequently, if the \( z \)-point, started from \( x_0, x_2, \ldots, x_n, \) always followed a particular path, then after change of \( h \) it can follow a different path even though started from the same \( z \) point.

(a) The hypothesis demands absoluteness. (b) The "reorganization" hypothesis demands step-functions, as we have shown above. (c) As we are not concerned with the possibility that the nervous system is limited in its variability of behavior, we may assume this to be infinite in the absence of any reason to the contrary.

Since a commutive system has an infinite number of \( z \)-fields, it clearly cannot be handled with the explicitness of equations (1). Assuming we have in front of us a real, physical example of a commutive system, we can clearly no longer know (a) the number of \( h \)'s, (b) their values, (c) the forms of the \( f \)'s in \( f_i(x_1, \ldots, h_i, \ldots) \), (d) the \( z \)-fields, since these depend on the \( f \)'s. But, and this is sufficient for the rest of the paper, (a) we can observe the values and changes of the \( x \)'s, (b) we can control the \( x \)'s starting point, and (c) we can test for constancy of the \( h \)'s by seeing whether the \( x \)'s path in the \( z \)-space repeats itself.

The disturbances are needed to test for, and demonstrate, equilibrium.

It is not necessary, for there are other ways of getting \( h \)-constancy, depending on some relation between the distribution of \( e \)-points and the direction of paths in the fields. We have no right to say that these combinations cannot occur.

Some minor postulate that the disturbance is to return the \( z \)-point to \( A, B, \ldots \) by some definite route devoid of \( e \)-points is required here.

Firstly, by selecting special examples it is easy to show that "equilibrium" belongs strictly to a single path and not to a field. Other special cases show that the path is to be confined to a region and not necessarily to terminate at a point. It may easily be shown that the common examples of equilibrium are all special cases, or limits, of the definition given. Nothing less general seems to be adequate.

The \( z \)-point is first to be fixed, but \( p \) is not to depend on where it is fixed. "At least" because, as shown in Note 9, other ways of getting \( h \)-constancy are possible, though rare and of little importance. \( p \) does not include these, so their presence may result in an increase of the proportion which have become \( h \)-constant.

This does not apply to a single field, i.e., the "commutive" part is necessary; for in a single field, as \( t \to \infty \), \( P \) does not \( \to 1 \) but \( \to \, \text{p} \). If \( p \) is small the effect is quite different.

The hypotheses about \( p \) and \( m \) are really only that (1) \( p \neq 0 \) and (2) \( m \to \infty \) as \( t \to \infty \). These merely exclude peculiar cases.

If the \( e \)-points are distributed systematically, the paths of equilibrium will automatically have systematic properties imposed on them. This leads to interesting and important developments which have already been explored by the author, but they are outside the scope of this paper.

The above discussion deals with one system where the \( z \)-point either is, or is not, on an equilibrail path, and the equilibrium is therefore "all or none." But there are several ways in which we can get independent equilibria, and then, following the same principles as before, the number of equilibria must tend to increase by accumulation. But this leads beyond the scope of the present paper.

We have supposed the environment to remain constant as far as its organization is concerned (though not the values of its variables). This means that the environment can interact freely with the nervous system in the above theory. A single change of the environmental organization would, however, wreck an established equilibrium. But regular changes between a finite set of environments can be shown to tend to equilibrium. If the environment should change its organization irregularly, the whole paper becomes inapplicable since postulate (b) of § 1 is no longer true.